

On detecting Higgs coupling in transitions of light atoms

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In light of the known Higgs mass and the current constraints on the quark-lepton Higgs coupling, we derive conditions for extracting upper limits on the lepton-nucleon Higgs coupling from light atoms and ions, assuming the availability of locally precise two- and three-body methods might be beneficial. A recent work has proposed to extract these limits in heavy atoms where the Higgs term is enhanced by $\approx 10^3 AZ$, due to both the large coupling modifier and large A , Z , and assuming sufficiently precise relativistic electron wave functions. We first revisit the old idea of using the Lamb shift in light muonic ions where the coupling is enhanced by about $201^3 AZ^3$ primarily due to the concentration of the muon wave function at the origin, the muon coupling modifier already being close to 1. For the muonic helium an experimental precision below 0.1 ppm is required to reach the constraints on Higgs couplings. However, theoretical uncertainty is large due to nuclear potential dependence of the finite size terms enhanced by the small muon orbit, and their elimination by using several states is precluded due to the Lamb shift being the only precisely measurable state. In normal (electronic) light systems transitions between low-lying states lie near the optical region allowing precise experiments, and extraction may be possible by eliminating the finite-size, polarization and Zemach moment terms from a set of transitions, e.g. $1S-2S$ and improved 2^3S-2^3P and 2^1S-2^3S in He^+ , while isotope shifts could be used if additional transitions are measured as precisely.

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I. INTRODUCTION

There has been a proposal to extract limits on Higgs nucleon-lepton coupling constant from valence electron transitions in heavy atoms [1–3]. In the present paper we examine the conditions for extracting these limits from transitions in light muonic or normal atoms or ions. The advantages and disadvantages of the two proposals are as follows.

The heavy-atom proposal [1–3] is based on the coupling enhancement due to the large atomic number A ; the stability of systems with large A allowing the experiment to reach precision of the order of 1 Hz in atomic clock transitions; and the relatively small other corrections like the weak force, despite the fact that it may mask the Higgs contribution. Due to the large number of precisely measurable transitions available, uncertainties in theoretical corrections depending on total charge Z can be conveniently avoided [1, 2] by the use of isotope shifts, i.e. the deviations from linearity in the King’s plots [4], as the change in A affects the transitions via nuclear recoil, electron correlations and nuclear charge radius independently of the transition measured. Disadvantages are the approximate nature of the factorization of the screened, relativistic electron wave function entering the transition matrix element, and the reliance on the existence of new physics via the relatively large current value (of the order of 10^3) of the coupling modifier κ_e for the electron-Higgs coupling constant y_e relative to its Standard Model (SM) value, $y_e = \kappa_e y_e^{\text{SM}}$.

The light-system proposal seems attractive as even

for the three-body system $e\mu^4\text{He}$ locally precise nonrelativistic wave functions can be calculated by e.g. the Correlation-function Hyperspherical Harmonic Method (CFHHM) [5], while for two-body ions like $\mu^4\text{He}^+$ both relativistic and non-perturbative methods are available. (In heavier muonic systems, electron screening of muonic orbits remains weak but scaling of nuclear structure effects strongly amplifies theoretical uncertainties.)

We start by applying the known Higgs mass to the old estimates [6], which used a much smaller mass, of the Higgs contribution to the muonic ^4He Lamb shift. This is measured by the CREMA collaboration [7] to a 10^{-5} level using laser spectroscopy. Different muon and electron reduced masses resulting in small muonic orbits provide a large coupling enhancement $(m_{\mu A}/m_{eA})^3$. A disadvantage is the $2S$ state lifetime of the order of $1\ \mu\text{s}$ [8, 9], a result of the finite muon lifetime of $2.2\ \mu\text{s}$, the $1S-2S$ two photon decay time of $8\ \mu\text{s}$ and the collisional quenching rate in gas. It prevents detecting effects below 1 MHz (10^{-6} meV) and complicates preparation of states. Another disadvantage is that while the sum of QED corrections (1813.02 meV for $\mu^4\text{He}^+$ [6]) is well known and can be improved, the finite nuclear size and polarization corrections also scale with the lepton mass, amplifying the uncertainties in the nuclear charge radii.

Next we look at normal (electronic) light systems where the precisely measured transitions (near the optical region) are those between low-lying nS states, losing the enhancement by the muon but benefitting from smaller nuclear structure corrections and measurements at the $10^{-12} - 10^{-15}$ level, related to the extraction of electronic charge radii [7] and the Rydberg constant.

A common disadvantage of light systems is that the perturbation theory in (αZ) , where α is the fine structure constant, has been well studied only up to $(\alpha Z)^6$,

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as appropriate for charge radii extraction [10, 11]. For muonic systems the nonrelativistic and relativistic expansions give identical results [10] while for electronic systems relativistic treatment is required [12, 13].

In the next section we calculate the current orders of magnitude of the Higgs coupling parameters for light systems. This is followed by an overview of the current light ion physics and the bottlenecks for reducing its uncertainty. The last section gives requirements on theoretical terms appearing in the expression for the coupling constant based on the isotope shifts when only two transitions are available.

II. BOUNDS ON THE HIGGS TERM

Higgs exchange between a nucleus and a bound electron or muon results in a potential of the Yukawa type,

$$V_H(r) = -g_{H\mu A} \frac{e^{-m_H r}}{r}, \quad (1)$$

where $g_{H\mu A}$ is proportional to the muon and nuclear coupling constants,

$$g_{H\mu A} = \frac{y_\mu y_A}{4\pi}. \quad (2)$$

The SM fermion-Higgs coupling constants are proportional to the fermion (F) mass according to the assumed hierarchy leading to fermion masses, as well as to the coupling modifiers based on experimental upper bounds on couplings which allow for the existence of new physics:

$$y_F = \kappa_F \frac{m_F}{v}, \quad v = 246 \text{ GeV}. \quad (3)$$

For the electron,

$$y_e = \kappa_e \times 2.1 \times 10^{-6}. \quad (4)$$

Ref. [1] uses y_e at the upper bound set by the LHC data on $H \rightarrow e^+e^-$ [14–16], where $\kappa_e < 611$ [16],

$$y_e < 611 \times 2.1 \times 10^{-6} \approx 1.3 \times 10^{-3}. \quad (5)$$

This κ_e value corresponds to the lowest new physics scale, 5.8 TeV [16], but the bounds are likely to improve, lowering the y_e value and reducing the feasibility of the proposal [1, 2].

For the muon, $\kappa_\mu = 0.2_{-0.2}^{+1.2}$ in Table 15 of Ref. [17] so we do not get the advantage of a weak experimental upper bound, resulting practically in the SM coupling value,

$$y_\mu \lesssim 1.4 \times 207 \times y_e^{\text{SM}} \approx 0.6 \times 10^{-3}, \quad (6)$$

using the upper bound of κ_μ .

The nuclear coupling is approximately proportional to the atomic number A ,

$$y_A = (A - Z)y_n + Zy_p \approx Ay_N, \quad (7)$$

where $y_n \approx y_p \approx y_N$ are the neutron and proton coupling constants which are linear combinations of the quark and gluon coupling constants. In more detail [1, 18–21] and neglecting the c_g term [22],

$$\begin{aligned} y_n &\approx 7.7y_u + 9.4y_d + 0.75y_s, \\ y_p &\approx 11y_u + 6.5y_d + 0.75y_s. \end{aligned} \quad (8)$$

The weakest bounds on individual quark couplings are $y_q \lesssim 0.3$ [23–25], where y_q is one of y_u , y_d , y_s , or y_c , resulting in $y_N \lesssim 3$ due to suppression of light quarks. LHC and electroweak data give a medium bound $y_q \lesssim 1.6 \times 10^{-2}$ [24, 26, 27], resulting in $y_N \lesssim 0.2$. Indirect bounds may be even lower, $y_q \lesssim 5 \times 10^{-3}$ [1, 28] resulting in $y_N \lesssim 10^{-3}$. These results translate to the following range of current upper bounds on the nuclear coupling y_A in muonic hydrogen,

$$y_1 \approx \{10^{-3}, 0.2, 3\}, \quad (9)$$

and in muonic ^4He :

$$y_4 \approx 4 \times \{10^{-3}, 0.2, 3\}. \quad (10)$$

The corresponding bounds on $g_{H\mu A}$ are linear in A :

$$g_{H\mu 1} \lesssim \{0.5 \times 10^{-7}, 1 \times 10^{-5}, 0.1 \times 10^{-3}\} \quad (11)$$

for μH and

$$g_{H\mu 4} \lesssim \{2 \times 10^{-7}, 4 \times 10^{-5}, 0.6 \times 10^{-3}\} \quad (12)$$

for $\mu^4\text{He}^+/e\mu^4\text{He}$.

Due to the large Higgs mass, the Higgs term is given by the leading order of perturbation in $(1/m_H)^2$, unlike Ref. [6] where m_H ranged from 0.15 MeV to 750 MeV. It is negligible in states with orbital angular momentum $l > 0$. For the Lamb shift, at the principal quantum number $n = 2$, the Higgs term in the transition energy ΔE_{LS} comes from the $2S$ state matrix element,

$$\delta_H(\Delta E_{LS}) \approx g_{H\mu A} |R_{20}(0)|^2 \frac{1}{m_H^2}, \quad (13)$$

where R_{20} is the $n = 2$, $l = 0$ radial wave function R_{nl} of the lepton x ,

$$|R_{n0}(0)|^2 = 4 \left(\frac{\alpha Z m_{xA}}{n} \right)^3. \quad (14)$$

In general, for transition i ,

$$\delta_H(\Delta E_i) \approx g_{HxA} \frac{4}{m_H^2} \left(\alpha Z m_{xA} \right)^3 \Delta_i, \quad (15)$$

where $\Delta_i = \delta_{l_{i1,0}}/n_{i1}^3 - \delta_{l_{i2,0}}/n_{i2}^3$.

The 201^3 -fold enhancement of the muonic Higgs term relative to electronic transitions at the same n due to the smaller muon reduced Bohr radius together with the $\approx AZ^3$ scaling results in total enhancement $\approx 201^3 \times AZ^3$.

For comparison, the coupling in the proposal [1–3] is enhanced relative to single nucleon-electron coupling through y_A by way of large A , and through the electron wave function squared at origin, $|\psi_e(0)|^2$, by way of large Z (screening in heavy atoms precludes enhancement by Z^3 by an unperturbed electron wave function, resulting in enhancement by Z instead [1]). At the current coupling modifier their total enhancement is $\approx 611 \times AZ$.

TABLE I: Requirements for extracting the Higgs contribution $\delta_H(\Delta E_{LS})$ from the Lamb shift of $\mu^4\text{He}^+$ and μH for the known Higgs mass (except row 5, see below). $\delta_{H\nu}$ are the corresponding frequencies. Bounds on the nucleon-Higgs coupling y_A are from Ref. [1]. $\eta_H = 10^6 |\delta_H(\Delta E_{LS})/\Delta E_{LS}|$ is the required precision in ppm. Rows 4 and 5 correspond to the precision defined as the discrepancy between theory and experiment in Ref. [6]: row 4 shows the extrapolated $g_{H\mu A}$ needed for observation of 125 GeV Higgs at that η_H , and row 5 is for an old 200 ppm experiment using the Higgs mass 0.75 GeV as discussed in Ref. [6] also for lower masses. The μH result (row 6) is for upper bound on Higgs coupling.

| system | ΔE_{LS} (meV) | $g_{H\mu A}$ | $\delta_H(\Delta E_{LS})$ (meV) | $\delta_{H\nu}$ (Hz) | η_H (ppm) |
|--------------------|--------------------------|----------------------|------------------------------------|-------------------------|--------------------|
| $\mu^4\text{He}^+$ | 1664 | 2×10^{-7} | 2×10^{-8} | 5×10^3 | 10^{-5} |
| | | 4×10^{-5} | 4×10^{-6} | 1×10^6 | 3×10^{-3} |
| | | 0.6×10^{-3} | 0.6×10^{-4} | 2×10^7 | < 0.1 |
| | | 36 | 4 | 1.0×10^{12} | 3×10^3 |
| | | 1.3×10^{-3} | 4 | 1.0×10^{12} | 3×10^3 |
| μH | 202 | 0.1×10^{-3} | 1×10^{-6} | 3×10^5 | < 0.01 |

Table I gives experimental accuracy requirements for extracting the Higgs term from Lamb shift for $\mu^4\text{He}^+$ or $e\mu^4\text{He}$ and μH . Errors due to uncertainties of the fundamental constants are of the order of 10^{-5} meV [29]. The Higgs term lies above the muon decay limit and within the upper range of $g_{H\mu A}$ only in muonic helium or heavier systems. The Z^3 transition energy scaling also favors heavier systems as the required relative accuracy is smaller.

For fixed coupling modifiers, the Higgs coupling scales to the normal (electronic) light systems as $g_{HeA} \approx 2.2 g_{H\mu A}$ for the same A , n , and the Higgs term scales as $g_{HeA} (m_{eA}/m_{\mu A})^3 \approx 0.27 \times 10^{-6}$.

TABLE II: As in Table I, but for the normal (electronic) systems and for the corresponding upper bounds (proportional to A) on the Higgs coupling; relative accuracy is omitted.

| system | transition | ΔE (meV) | g_{HeA} | $\delta_H(\Delta E)$ (meV) | $\delta_{H\nu}$ (Hz) |
|-----------------|---------------|---------------------|----------------------|-------------------------------|-------------------------|
| $^4\text{He}^+$ | Lamb shift | 0.05 | 1.3×10^{-3} | 2×10^{-11} | 4 |
| | $2^3S - 2^3P$ | 1.15×10^3 | | 2×10^{-11} | 4 |
| | $1S - 2S$ | 40×10^3 | | 1×10^{-10} | 30 |
| H | $1S - 2S$ | 10×10^3 | 3×10^{-4} | 3×10^{-12} | 0.8 |

In electronic systems the Lamb shift is not in the optical range and not very precisely measured. In the helium ion it is $14 \text{ GHz} \pm 348 \text{ kHz}$ or 0.05 meV [30] (Table II), the Higgs term is 4 Hz ($1.7 \times 10^{-11} \text{ meV}$) at saturated coupling, and the uncertainty would have to be decreased by 10^5 , more than in muonic systems. Better precision is achieved in the near-optical transitions, the centroid $2^3S - 2^3P$ transition having 2.4 kHz uncertainty (10^{-8} meV) [31], and in the $1S - 2S$ transition [30]. For comparison we give the hydrogen $1S - 2S$ transition [7].

The current uncertainties in the above transitions and the required increase in precision is discussed in the next section.

III. CURRENT UNCERTAINTIES

Experiments in light systems focus on the extraction of the charge radii and the Rydberg constant. We identify suitable transitions for Higgs term extraction and the bottlenecks for reducing their uncertainties.

The $2S_{1/2} - 2P_{3/2}$ and $2S_{1/2} - 2P_{1/2}$ transitions used for calculating [32, 33] the $\mu^4\text{He}^+$ Lamb shift were measured long ago at about 1528 meV and 1381 meV, respectively [34, 35]. At the time of the proposal [6] based on the light Higgs, the discrepancy between theory and experiment as given by the uncertainty of the finite-size ($-288.9 \pm 4.1 \text{ meV}$) and nuclear polarization terms ($3.1 \pm 0.6 \text{ meV}$) was an order of magnitude larger than the experimental uncertainties, $\pm 0.3 \text{ meV}$ ($\pm 0.5 \text{ meV}$) or 200 ppm (330 ppm), respectively. (The electron scattering ^4He radius used was $1.674 \pm 0.012 \text{ fm}$ [36].)

Improved calculations [29] reduced the uncertainty of the non-nuclear contributions to the Lamb shift to 10^{-3} meV (240 MHz, 0.6 ppm) for $\mu^4\text{He}^+$, but that of the finite-size correction ($-295.848 \pm 2.8 \text{ meV}$) was still large (relative error 1×10^{-2}), double the error of the charge radius [29], and that of the nuclear polarization term of the two-photon exchange correction remained 0.6 meV. The uncertainty of the ^4He charge radius was $1.676(8) \text{ fm}$ (5×10^{-3} relative error) [37, 38]. The charge radius puzzle in the proton [39–41] spurred new measurements; it has recently been confirmed in μD [42]. The electron scattering ^4He charge radius is known to 2×10^{-3} accuracy ($1.681 \pm 0.004 \text{ fm}$) [43, 44].

As the current Lamb shift experiments aim to resolve the charge radius problem [7, 45], theoretical work is dedicated solely to improving the polarization terms but not the finite-size terms. (The complementary measurements of the electronic $1S - 2S$ transition in $^4\text{He}^+$ serve to test the QED part [30].) Laser spectroscopy of $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$ at “moderate” precision of 50 ppm [8, 43] yielded the charge radius to 1×10^{-3} accuracy [8]. The 2013–2014 $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$ 40 ppm measurements [9, 46] yielded the ^4He and ^3He charge radii to about 3×10^{-4} [9].

The transitions actually measured for the Lamb shift [33] are $2S_{1/2} - 2P_{3/2}$ and $2S_{1/2} - 2P_{1/2}$ in $\mu^4\text{He}^+$; the six transitions between the HFS-split $2S$ and $2P$ states

planned in Ref. [8, 43] in $\mu^3\text{He}^+$; and the $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}$ [9, 32, 33] in μH . These are combined with the theoretical $2P_{3/2} - 2P_{1/2}$ fine splittings and with the hyperfine splitting (HFS) of the ground state for nonzero-spin nuclei, or the electron-muon HFS for high precision [5]. The latter also allows the determination of the Zemach radius from the nuclear polarization or vice versa. The HFS precision, 4465.004(29) MHz (1.8×10^{-3} meV), was 6.5 ppm [47], while the CREMA collaboration is aiming at 1 ppm [9]. (These experiments are also used to check some terms in the Lamb shift: a contribution from the nuclear polarization term cancels the third Zemach moment $\langle r^3 \rangle_{(2)}$ in $\mu^4\text{He}^+$ [48] like it was observed earlier for μD [49, 50].) The current results are reviewed in Ref. [11] up to $\mu^4\text{He}^+$ and in Ref. [51] for μH . The vacuum polarization (VP) term contains the following uncertainties [11, 52–56]. The relativistic perturbative Uehling term in μH is modified by finite-size effects by about 0.0079 meV and 0.0082 meV for the two proton radii involved in the proton radius problem, 0.842 fm and 0.875 fm, respectively. The $\mu^4\text{He}^+$ finite-size effect in VP is $-0.3297 \langle r_\alpha^2 \rangle$ meV fm $^{-2}$ implying a similar uncertainty of 0.0016 meV for the current 5×10^{-3} ^4He radius uncertainty. Neglecting finite nuclear size in muon-electron VP causes moderately increasing shifts of up to 0.0001 meV for $\mu^4\text{He}^+$. Uncertainties of the “light-by-light” corrections reach 0.0006 meV for $\mu^4\text{He}^+$, while the sixth-order VP uncertainties reach 0.003 meV. The largest uncertainty within the VP terms is the hadronic VP, reaching an estimated 5% uncertainty in the 0.225 meV value for $\mu^4\text{He}^+$, or estimated 0.012 meV [11]. The relativistic recoil amounts to about 0.001 meV [52–56].

To return to the nuclear structure terms, the finite-size term proportional to the charge radius squared $\langle r_p^2 \rangle$ in μH has uncertainty 0.064 meV for the spectroscopic radius 0.875 fm and 0.010 meV for the Lamb shift radius 0.842 fm; however, the corresponding values -3.978 meV and -3.6855 meV differ by 8%, or 0.3 meV. The situation in $\mu^4\text{He}^+$ is worse due to the scaled contribution, amounting to 1.4 – 2.8 meV uncertainty depending on which radius is taken [11, 29].

The Lamb shift is usually parametrized in terms of charge distribution moments as $\mathcal{A} + \mathcal{B}\langle r^2 \rangle + \mathcal{C}(\langle r^2 \rangle)^{3/2}$ which is suitable at current precision; it also has the consequence that the measured Lamb shift itself is rarely quoted [57]. For $\mu^4\text{He}^+$, $\mathcal{B} = -106.344$ meV fm $^{-2}$ [11], and consists of six contributions, the largest being the leading term

$$b_a = -\frac{2\alpha Z}{3} \left(\frac{\alpha Z m_{\mu A}}{n} \right)^3 \quad (16)$$

in Eq. (5) of Ref. [10], amounting to $-105.319 \langle r_\alpha^2 \rangle$ meV fm $^{-2}$. (The total includes the finite-size VP correction $-0.3297 \langle r_\alpha^2 \rangle$ meV fm $^{-2}$ quoted above.) In $^4\text{He}^+$, the nonleading contributions $b_b + \dots + b_e$ amount to about a percent. Some depend on the assumed analytic charge distribution via terms e.g. $b_a(\alpha Z)^2 \langle \ln(\alpha Z m_{\mu A} r) \rangle$ (Ref. [11], Appendix B, and Ref. [10]). \mathcal{C} involves a model-

dependent transformation between the third Zemach moment $\langle r^3 \rangle_{(2)}$ and $(\langle r^2 \rangle)^{3/2}$ via a factor f_{Zem} which for $\mu^4\text{He}$ is about 3.5, but depends on the charge distribution already on the second digit [11]. In $\mu^4\text{He}$ the uncertainty of the \mathcal{C} term is one-third the uncertainty of the polarization term. The relativistic corrections start at $(\alpha Z)^6$ as verified in Ref. [10] using perturbation theory based on both the Schrödinger and the Dirac wave functions. Radius-independent corrections for μH are summarized in Table 1 of Ref. [51].

Reduction of the charge radius uncertainty to the level of the parameter dependence of the effective nuclear potentials has been achieved in recent *ab-initio* calculations of the inelastic term in the two-photon exchange correction in light muonic atoms using state of the art nuclear potentials (AV18 and χEFT) and the hyperspherical harmonic EIH method [48, 58, 59]. The nuclear problem was solved separately and the polarization terms evaluated in second-order perturbation theory in terms of the residual Coulomb potential for point nucleons. A 5×10^{-2} accuracy is required for determining the ^3He and ^4He charge radii squared to 3×10^{-4} [9, 43, 48, 58], ensuring the same absolute errors in both terms. The new value of the nuclear polarization term $-2.47(14)$ meV [48] has 6×10^{-2} accuracy (absolute error is misquoted as 0.015 meV in Ref. [11]) compared with the old [6] value 3.1 ± 0.6 meV (2×10^{-1} accuracy). The AV18 and χEFT potentials are tuned to the ^3He binding energy but they give different charge radii. Uncertainty of the polarization term may be further reduced using the ^4He charge radius to constrain the nuclear potential models [48], but beyond that any improvement seems unlikely.

TABLE III: Coefficients of the μH Lamb shift parametrization $\mathcal{A} + \mathcal{B}\langle r_p^2 \rangle + \mathcal{C}(\langle r_p^2 \rangle)^{3/2}$ for perturbative [11] and nonperturbative calculations. Higher terms from Ref. [60] are not quoted.

| Ref. | \mathcal{A} (meV) | \mathcal{B} (meV fm $^{-2}$) | \mathcal{C} (meV fm $^{-3/2}$) |
|------|---------------------|---------------------------------|-----------------------------------|
| [11] | 206.0611(60) | -5.22718 | 0.0365(18) |
| [61] | 206.0604 | -5.2794 | 0.0546 |
| [60] | 206.0465137 | -5.226988356 | 0.03530609322 |

Current nonperturbative calculations also suffer from the nuclear model dependence via the assumed charge distributions. They have been performed for muonic hydrogen (and could be extended to helium). The work [61] solves the Dirac equation to 500 neV accuracy but describes recoil only via the reduced mass of the muon leaving further corrections to perturbation theory. The charge distributions were given in terms of moments, i.e. $\langle r_p^2 \rangle$, to express results in the conventional form. Also, a number of corrections were not calculated [11] (two- and three-loop VP, muon self-energy, muon and hadron VP, and nuclear polarization). The terms differ from the perturbation theory to the order of 0.03 meV. The work [60] using the proton dipole form factor, Gaussian, uniform, Fermi and experimentally fitted charge distribu-

tions seems better converged, listing the calculated terms to better than 0.001 meV accuracy, but the charge distribution dependence was of the order of 0.004 meV in the Coulomb and VP terms. Methods are compared in Table III, with the differences up to 0.05 meV.

In summary, the uncertainty of the nuclear structure terms in light muonic systems is 3–4 orders of magnitude larger than the requirements in Table I and cannot be reduced further. The Zemach moment term has about 0.2 meV uncertainty in muonic helium [11].

In the normal (electronic) light systems the precision is higher and corrections must be based on the Dirac wave functions. The extensive literature [7, 12, 30, 31, 62, 63] is reviewed e.g. in CODATA [64].

The highest precision is achieved in H which is less favorable for Higgs term extraction than He (Table II). The $1S - 2S$ transition in hydrogen used for deducing the Rydberg constant is currently measurable with 4×10^{-15} (1 Hz) uncertainty [7]. This almost meets the requirement of Table II, but it is overshadowed by the uncertainty of the nuclear structure corrections, as the relative size of the nuclear contributions to transition energy is 4×10^{-10} , or about 1 MHz, therefore they should be known to better than 6 places for direct extraction of the Higgs term, clearly not achievable as the charge radius uncertainty (a decade ago) was 44 kHz and the B_{60} and B_{7i} terms of the two-loop QED corrections [65] are -8 kHz [30].

In $^4\text{He}^+$, The $1S - 2S$ transition at 9.9×10^{16} Hz is already predictable with 0.35 MHz uncertainty which is 4 orders of magnitude worse than the requirement of Table II [30], the largest uncertainties stemming from the charge radius and the B_{60} and B_{7i} terms. The accuracy of the $1S - 2S$ transition could be improved to 10^{-16} (1 Hz) [30], which would be 30 times better than the requirement of Table II. Other suitable transitions, for example the $2^3S - 2^3P$ at 1.1 eV, are currently at the 10^{-10} uncertainty or 2 kHz level [31], which is 3 orders of magnitude short of requirements of Table II. The nuclear structure contributions are amplified with respect to hydrogen. They are given in detail in Ref. [63]. The nonlogarithmic relativistic correction f_{fs} depends on the assumed nuclear charge distribution model in the leading digit, and its relative contribution $(Z\alpha)^2 f_{\text{fs}}$ to the finite nuclear size term is 4×10^{-5} .

In transitions between $2S$ and $2P$ states of the normal (electronic) helium the finite-size terms, which scale as $Z^4 m_{xA}^3$, can be estimated to be at the 0.4×10^{-4} meV (9 MHz) level so direct extraction of the Higgs terms is ruled out here as well. The Zemach moment term is 4×10^{-5} times the finite-size term [63], or 1.6×10^{-9} meV, so a typical one percent nuclear uncertainty would appear at the 10^{-11} meV level. This is close to the requirement of Table II so this term cannot be a priori excluded. (The uncertainty in the Rydberg constant, known to 2×10^{-11} [7], cancels out in the Higgs extraction.)

IV. EXTRACTION OF THE HIGGS TERM

In normal heavy atoms of proposal [1–3] isotope shifts are the most promising method of extracting the Higgs term, by looking for the departure from the linearity of the King's plots [2, 4] for a pair of transitions. For large A, A' , the coefficient of $\langle r^2 \rangle_A - \langle r^2 \rangle_{A'}$ in the $A - A'$ isotope shift is essentially independent of A, A' . For example, the relative isotope shift of electron reduced masses for $A = 100, A' = 101$ is 5×10^{-8} and that of the leading term of the finite-size coefficient is three times that. A similar argument regarding the King's plot linearity is made in Ref. [1]. Using isotope shifts of two measured transitions we can eliminate the $\langle r^2 \rangle_A - \langle r^2 \rangle_{A'}$ terms. The validity across several isotope pairs A, A' of the resulting linear relation between the two isotope shifts can then be studied. This requires at least two transitions measured for three different A , but there are many suitable transitions in heavy atoms.

In light systems we have a similar parametrization of transition energies but with the additional term (Zemach moment term) $C' \langle r^3 \rangle_{(2)}$ that needs to be eliminated together with $\mathcal{B} \langle r^2 \rangle$ (the Zemach moment has to be used to avoid the model-dependent transformation factor f_{Zem} [11, 63]). To reduce the number of required transitions, one could presumably do this without isotope shifts using three transitions ($i = 1, 2, 3$) of a fixed isotope, provided $\mathcal{A}_i, \mathcal{B}_i, C'_i$ are known:

$$\Delta E_i = \mathcal{A}_i + \mathcal{B}_i \langle r^2 \rangle + C'_i \langle r^3 \rangle_{(2)} + c \mathcal{H}_i \quad (17)$$

where

$$c = g_{H\mu 1} = \frac{y_\mu y_N}{4\pi} < 3 \times 10^{-4}, \quad (18)$$

$$\mathcal{H}_i = \frac{4}{m_H^2} \left(\alpha Z m_{xA} \right)^3 \Delta_i. \quad (19)$$

(We leave out the weak interaction term [1, 2].) Assuming for simplicity that $\Delta_3 = 0$ (see below),

$$c = \frac{1}{A} \frac{(e_1 \mathcal{B}_{23} - e_2 \mathcal{B}_{13}) - (C'_1 \mathcal{B}_2 - C'_2 \mathcal{B}_1) e_3 / C'_3}{\mathcal{H}_1 \mathcal{B}_{23} - \mathcal{H}_2 \mathcal{B}_{13}} \quad (20)$$

where

$$e_i = E_i - \mathcal{A}_i, \quad \mathcal{B}_{ij} = \mathcal{B}_i - \frac{C'_i}{C'_j} \mathcal{B}_j. \quad (21)$$

In this case measurements on a different isotope A' if available would be used independently to improve c .

Isotope shifts in light systems introduce yet more terms requiring more transitions to eliminate them. The isotope shift of the leading contribution b_a (Eq. (16)) to \mathcal{B} is 3 percent for $A = 3, A' = 4$ due to muon reduced mass shift. b_b, b_c, \dots [11] also depend on A via $m_{\mu A}$ starting at order $(\alpha Z)^6$. In electronic light systems the relative isotope shift of \mathcal{B} is 10^{-4} (negligible [11] for the charge radius determination but not for the Higgs extraction). If we denote the isotope shift of a by $[a]_{AA'} = a_A - a_{A'}$,

we have for the transition energy ΔE_i (leaving out the AA' suffix for brevity):

$$[\Delta E_i] = [\mathcal{A}_i] + \mathcal{B}_i[\langle r^2 \rangle] + [\mathcal{B}_i]\langle r^2 \rangle_{A'} + \mathcal{C}'_i[\langle r^2 \rangle^{3/2}] + [\mathcal{C}'_i]\langle r^3 \rangle_{(2)A'} + c[\mathcal{A}\mathcal{H}_i]. \quad (22)$$

The term corresponding to $[\mathcal{B}_i]_{AA'}\langle r^2 \rangle_{A'}$ is claimed sufficiently small for heavy atoms [1]. The Zemach moment term $([\mathcal{C}'_i]_{AA'}\langle r^3 \rangle_{(2)A'})$ may also turn out to be small enough (e.g. in ${}^4\text{He}^+$, above), permitting the use of four instead of five transitions. Obviously we cannot look for King's plot-type linearity here as we are likely to have only 2 isotopes. Also, and the coefficients \mathcal{B}_i and \mathcal{C}'_i may depend on A appreciably in higher orders of (αZ) .

In μHe , even if the precision of the Lamb shift proper could be improved (Table I), there is no suitable second transition; the $1S - 2S$ (or the appropriate centroid energy [12, 63]) at 8 keV lies in the X-ray region where the experimental precision is smaller but the required relative precision is 10^3 times larger than for the Lamb shift. (We cannot take the two transitions to be a pair of the separate transitions measured for the Lamb shift as their Higgs terms cancel and, for isotope shifts, quantum numbers have no counterparts between $A = 4$, $A' = 3$.)

In ${}^4\text{He}^+$ measurements of sufficient precision for at least three transitions seem possible in principle as per above: 1 Hz accuracy has already been suggested for $1S - 2S$ [30], and we assume the missing 3 orders of magnitude improvement in $2^3S - 2^3P$ to be possible. These two transitions have nonvanishing Higgs terms which differ in Δ_i (Eq. (15)). Theoretical ${}^4\text{He} - {}^3\text{He}$ isotope shifts (with the precision required for extracting the charge radii) for both the $2^3S - 2^3P$ and for the $2^1S - 2^3S$ transition at 0.8 eV have been calculated [63]. The Higgs contribution vanishes in the latter but it provides the third equation (17), making it possible to eliminate the nuclear structure terms. It has been measured to 8×10^{-12} or 1.8 kHz [66]. Like in muonic experiments, current precision is geared to the extraction of charge radii from isotope shifts.

The denominator of Eq. (17) of course vanishes in the leading order in (αZ) . The A dependence of \mathcal{B}_i starts at order $(\alpha Z)^6$.

V. CONCLUSION

Current upper bounds on both the muon-nucleon and electron-nucleon Higgs coupling constrain the possibility of Higgs term extraction from the Lamb shift to muonic helium and heavier systems. The Z^4 scaling of the nuclear structure corrections, existing experimental work on light muonic systems, as well as more difficult control of the number of ejected electrons during the muon cascade in heavier systems, all favor lighter systems, making the muonic and electronic helium ion the preferred system. Direct extraction of the Higgs term is not viable because of too large nuclear structure terms, which exhibit uncertainties too large by 3 – 4 orders of magnitude. Due to their dependence on either the effective nuclear potentials or on assumed charge distributions, this uncertainty cannot be reduced. Instead, (i) availability of several transitions is required to eliminate these terms, and (ii) uncertainty has to be reduced a few orders of magnitude below the current which is geared towards extracting the charge radii. In muonic helium the Lamb shift experimental precision should be at least 0.1 ppm, possibly requiring the evaluation of small effects [67], but there is no other suitable, precisely measurable transition. The normal (electronic) helium ion is more promising, offering the lowest-lying transition measurable to 1 Hz accuracy and several states currently measured to kHz precision. We give required elevated precision (above that of the Rydberg constant) which may allow elimination of nuclear structure terms. The resolution of the charge radius puzzle which is the current focus of the light muonic and electronic ion physics does not require this level of precision.

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- [1] C. Delaunay, R. Ozeri, G. Perez and Y. Soreq, arXiv:1601.05087v1 [hep-ph] (2016).
 - [2] C. Delaunay and Y. Soreq, arXiv:1602.04838v1 [hep-ph] (2016).
 - [3] C. Frugiuele, E. Fuchs, G. Perez and M. Schlaffer, arXiv:1602.04822v1 [hep-ph] (2016).
 - [4] W.H. King, J. Opt. Soc. Am. **53**, 638 (1963).
 - [5] R. Krivec and V.B. Mandelzweig, Phys. Rev. A **56**, 3614 (1997), Phys. Rev. A **57**, 4976 (1998).
 - [6] E. Borie and G.A. Rinker, Rev. Mod. Phys. **54**, 67 (1982).
 - [7] A. Antognini, arXiv:1512.01765v2 [physics.atom-ph] (2016).
 - [8] T. Nebel et al., Hyperfine Int. **212**, 195 (2012).
 - [9] R. Pohl et al., arXiv:1609.03440v1 (2016).
 - [10] J.L. Friar, Ann. Phys. **122**, 151 (1979).
 - [11] E. Borie, Ann. Phys. **327**, 733 (2012); arXiv:1103.1772v7 (2014).
 - [12] V.A. Yerokhin, Phys. Rev. A **83**, 012507 (2011).
 - [13] A. Czarnecki, U.D. Jentschura and K. Pachucki, Phys. Rev. Lett. **95**, 180404 (2005).
 - [14] G. Aad et al. (ATLAS), Phys. Lett. **68**, B738 (2014).
 - [15] V. Khachatryan et al. (CMS Collaboration), Phys. Lett. B **744**, 184 (2015).
 - [16] W. Altmannshofer, J. Brod and M. Schmaltz, JHEP **05**,

- 125 (2015).
- [17] <http://cds.cern.ch/record/2052552/files/ATLAS-CONF-2015-044.pdf>, last accessed 2016/06/10.
- [18] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Lett. B **78**, 443 (1978).
- [19] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. **180**, 747 (2009).
- [20] P. Junnarkar and A. Walker-Loud, Phys. Rev. D **87**, 114510 (2013).
- [21] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. **185**, 960 (2014).
- [22] ATLAS and CMS (2015), ATLAS-CONF-2015-044.
- [23] V. Khachatryan et al. (CMS Collaboration), Eur. Phys. C **75**, 212 (2015).
- [24] G. Perez, Y. Soreq, E. Stamou and K. Tobioka, Phys. Rev. D **92**, 033016 (2015).
- [25] Y. Zhou, arXiv:1505.06369 (2015).
- [26] C. Delaunay, T. Golling, G. Perez and Y. Soreq, Phys. Rev. D **89**, 033014 (2014).
- [27] A.L. Kagan, G. Perez, F. Petriello, Y. Soreq, S. Stoynev and J. Zupan, Phys. Rev. Lett. **114**, 101802 (2015).
- [28] G. Perez, Y. Soreq, E. Stamou and K. Tobioka, Phys. Rev. D **93**, 013001 (2016).
- [29] A.P. Martynenko, Phys. Rev. A **76**, 012505 (2007).
- [30] M. Herrmann et al., Phys. Rev. A **79**, 052505 (2009).
- [31] P. Cancio Pastor et al., Phys. Rev. Lett. **108**, 143001 (2012).
- [32] C.E. Carlson, V. Nazaryan and K. Griffioen, Phys. Rev. A **83**, 042509 (2011).
- [33] C.E. Carlson, Progress in Particle and Nuclear Physics **82**, 59 (2015).
- [34] G. Carboni et al., Nucl. Phys. A **278**, 38 (1977).
- [35] G. Carboni et al., Phys. Lett. B **73**, 229 (1978).
- [36] I. Sick, J.S. McCarthy and R.R. Whitney, Phys. Lett. B **64**, 33 (1976).
- [37] J.L. Friar, Lect. Notes Phys. **627**, 59 (2003).
- [38] I. Sick, Phys. Lett. B **116**, 212 (1982).
- [39] R. Pohl, R. Gilman, G.A. Miller and K. Pachucki, Ann. Rev. Nucl. Part. Sci. **63**, 175 (2013); arXiv:1301.0905.
- [40] T.P. Gorringe and D.W. Hertzog, Progress in Particle and Nuclear Physics **84**, 73 (2015).
- [41] J.C. Bernauer et al., Phys. Rev. Letters **105**, 242001 (2010).
- [42] R. Pohl et al. (CREMA collaboration), Science **353**, 669 (2016).
- [43] A. Antognini et al., Can. J. Phys. **89**, 47 (2011).
- [44] I. Sick, Phys. Rev. C **77**, 041302 (2008).
- [45] R. Pohl for the CREMA collaboration, Journal of the Physical Society of Japan **85**, 091003 (2016).
- [46] A. Antognini et al., arXiv:1509.03235v2 (2015); Proc. 21st International Conference on Few-Body Problems in Physics, Chicago, USA, May 18-22, 2015, C. Elster, D.R. Phillips and C.D. Roberts (eds.).
- [47] C.J. Gardner et al., Phys. Rev. Lett. **48**, 1168 (1982).
- [48] C. Ji, N. Nevo Dinur, S. Bacca and N. Barnea, Phys. Rev. Letters **111**, 143402 (2013).
- [49] J.L. Friar, arXiv:1306.3269, Phys. Rev. C **88**, 034003 (2013).
- [50] K. Pachucki, Phys. Rev. Lett. **106**, 193007 (2011).
- [51] R. Pohl, A. Antognini, F. Nez, F.D. Amaro, F. Biraben et al., Nature (and Supplementary Material) **466**, 213 (2010).
- [52] S.G. Karshenboim, V.G. Ivanov, E.Yu. Korzinin and V. A. Shelyuto, Phys. Rev. A **81**, 060501 (2010).
- [53] S.G. Karshenboim, V.G. Ivanov and E.Yu. Korzinin, Phys. Rev. A **85**, 032509 (2012).
- [54] U.D. Jentschura, Phys. Rev. A **84**, 012505 (2011).
- [55] E.N. Elekina, A.A. Krutov and A.P. Martynenko, Phys. Part. Nucl. **8**, 331 (2011).
- [56] A.A. Krutov et al., JETP Lett. **120**, 73 (2015).
- [57] M. Diepold, PhD Dissertation, Ludwig-Maximilians-Universität München, Munich 2015.
- [58] Chen Ji, O.J. Hernandez, N. Nevo Dinur, S. Bacca and N. Barnea, arXiv:1509.01430v1 (2016).
- [59] O.J. Hernandez, N. Nevo Dinur, Chen Ji, S. Bacca and N. Barnea, arXiv:1604.06496v1 (2016).
- [60] P. Indelicato, Phys. Rev. A **87**, 022501 (2013).
- [61] J.D. Carroll, A.W. Thomas, J. Rafelski and G.A. Miller, Phys. Rev. A **84**, 012506 (2011); arXiv:1104.2971v3.
- [62] B. de Beauvoir et al., The European Physical J. **D**, 61 (2000).
- [63] K. Pachucki and V.A. Yerokhin, arXiv:1503.07727v2 (2015).
- [64] P.J. Mohr, B.N. Taylor, and D.B. Newell, Rev. Mod. Phys. **84**, 1527 (2012).
- [65] V.A. Yerokhin, P. Indelicato and V.M. Shabaev, arXiv:physics/0611265v1 (2006).
- [66] R. van Rooij et al., Science **333**, 196 (2011); arXiv:1105.4974v1.
- [67] P. Amaro et al., Phys. Rev. A **92**, 022514 (2015).